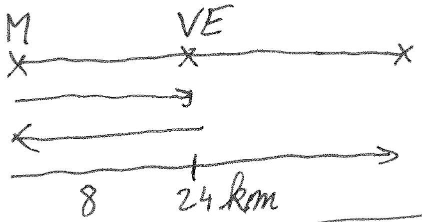
①



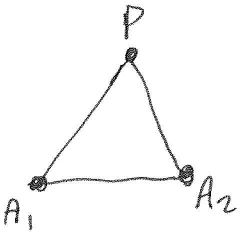
$\boxed{16\ km}$

②

0 : 1
1 : 2
2 : 4
3 : 8
4 : 16
5 : 32
6 : 50
7 : 68
8 : 86
9 : 104
10 : 122

$\longrightarrow \boxed{121}$

③



$75° - 30k°$
↓
30, 60, 90

~~105 - 30k~~

75	30	75	$\rightarrow 3 \times 2$
75	60	45	$\rightarrow 6 \times 2$
75	90	15	$\rightarrow 6 \times 2$

$\boxed{30}$

④

~~48~~

$48 = 2^4 . 3 =$

$1512 = 9 \times 3^3 \times 7 \times 2^3$

168
56

$840 = 5 \times 7 \times 2^3 \times 3$

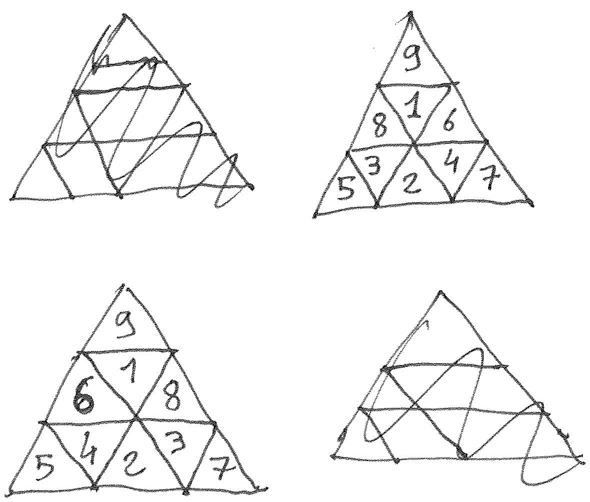
120
24

$1080 = 5 \times 9 \times$

120
24

$840 \rightarrow 2^3 . 3$
$1080 \rightarrow 2^3 . 3$
$1512 \rightarrow 2^3 . 3$

B C D



$24 = 1 \times 3 \times 8$
$= 1 \times 4 \times 6$
$= 2 \times 3 \times 4$

$\rightarrow 1, 3, 4$

$\boxed{6\ \text{solutions}}$

$\boxed{1}\ \boxed{6}\ 2 \times \boxed{4}\ \boxed{3} = 6\,9\,6\,6$

$+ \qquad - \qquad -$

$\boxed{89}\ n\ \boxed{82} + \boxed{1}\ p\ \boxed{6} = \boxed{1}\ \boxed{0}\ \boxed{8}\ q$

$= \qquad = \qquad =$

$\boxed{2}\ 5\ \boxed{4} \times \boxed{2}\ \boxed{7} = \boxed{6}\ \boxed{8}\ 5\ \boxed{8}$ $\qquad \boxed{6858}$

$\cancel{6}, 7, \cancel{8}$

$A = 0$ ou 1

$6966 = 9 \times 774 = 2 \times 9 \times 387 = 2 \times 3^4 \times 43$

$\square\square 2 = 2 \times 81 \qquad - 43 \qquad\qquad 2 = 2 \times 1$

$$\begin{array}{r} 162 \\ \times\ 43 \\ \hline 6966 \end{array}$$

$n + p = q \qquad n \geq 88 \qquad 107 \leq q \leq 116$

$p = q - n \leq 116 - 88 = 28 \qquad p \leq 28$

~~685x | 25y ; 1... | 2 ; 6850 | 250 ; 137 | 5~~

$686\cancel{0} / 25\cancel{0} =$

$\to 675/25 = 135/5 = \cancel{27} \to 28$ (max)

~~6850/260~~ $\quad 685/26$

~~342/343/13~~ $\quad 342/13 \to 26$ (min)

$$\begin{array}{r|l} 6850 & 26 \\ 165 & \overline{25} \end{array}$$

169×4

~~250×~~

$260 \times 26 \leq 170 \times 40 \leq 6800$

~~6850 | 27~~ $\qquad \underbrace{685}_{19+8}8$

$250 \times 28 = 7000$

$$\begin{array}{r} 254 \\ \times\ 27 \\ \hline 6858 \end{array}$$

6

9 - 7 - 7 - 9 $\boxed{19}$

7

~~A : -19 à 3~~
~~B :~~

$$\left.\begin{array}{l} a+b+c+d+e+f=0 \\ a+\quad c+d+e+f=2 \end{array}\right) \Rightarrow b=-2$$

$c+d+e+f=18 \Rightarrow a=-16$

$a+d+e+f=3 \Rightarrow d+e+f=19 \Rightarrow c=-1$

$e+f=17 \Rightarrow d=2$

~~a+b+c+e~~ $a+d+f=-3 \Rightarrow f=11$

$\Rightarrow e=\cancel{16}\ 6$

$P=+2\times-16\times2\times6\times11 \cancel{=}$ $\boxed{-4224}$

64 704 4224

$$\begin{array}{r} 176 \\ \times\ 24 \\ \hline 4224 \end{array}$$

8

~~Noyaux~~

~~k(n+1)~~ $k(n+1)+1=1993$

$n+1 \mid 1992$ $1992=2^3.3.83$

249

83

$n+1=83\times\begin{cases}1\\2\\3\end{cases}$

$\boxed{n=82,\ 165 \text{ ou } 248}$: 3 solutions.

9

x^2 —

84

$x^2 - 336 = y^2$

$y^2 - 336 = z^2$

$x^2 - y^2 = y^2 - z^2$

$\hat{m}$ parité ⇒ pairs.

$\alpha^2 - \beta^2 = 336 = (\alpha-\beta)(\alpha+\beta)$ α et β : $\hat{m}$ parité

$336 = 2^4 \times 3 \times 7$, $(\alpha-\beta) = \cancel{\ }\,4 \cdot \ldots$, $(\alpha+\beta) = 4 \cdot \ldots$ ← pairs.

~~q~~ ~~...~~

3 7 → $\begin{cases}\alpha = 4.5\\ \beta = 4.2\end{cases}$ ou $\begin{cases}\alpha = 4.11\\ \beta = 4.10\end{cases}$

1 21

$\boxed{20 \text{ et } 8}$ $\boxed{44 \text{ et } 40}$

$400 - 64 = 336$ $44^2 - 40^2 = 16(11^2 - 10^2) = 16 \times 21$

impairs : $\alpha = 2\alpha' + 1$, $\beta = 2\beta' + 1$

$4(\alpha'-\beta')(\alpha'+\beta'+1)$ $(\alpha'-\beta')(\alpha'+\beta'+1) = 2 \times 2 \times 3 \times 7$

4 21 → ~~...~~ 8 et 12 → 17 et 25

12 7 → 2 et 9 → 5 et 19 ×

28 3 → 12 et 15 → 25 et 31

84 1 → 41 et 42 → 83 et 85 ×

$31 \to 25 \to 17$

$961 \to 625 \to 289$

$\boxed{289}$

$$\begin{array}{r} 625 \\ \cancel{336} \\ \hline 961 \end{array} \qquad \begin{array}{r} 289 \\ 336 \\ \hline 625 \end{array}$$

$a + b = ab$

$(a-1)(b-1) = 1$

$a - 1 = \cancel{2^k}\ 2^k \cdot 5^l$ avec $|k| \leq 3$ et $|l| \leq 3$

$b - 1 = 2^{-k} \cdot 5^{-l}$

$\frac{1}{16}$

~~Si $a-1<0, b-1<0$~~

$a \geq 0 \Rightarrow a - 1 \geq -1$

$b - 1 \geq -1$

$(0;0)$, $(\underset{7}{k}; \underset{7}{l}) \rightarrow \boxed{50}$

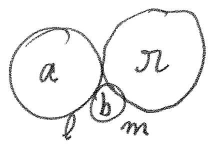
~~12~~ 11

~~a, b,~~ $3 \leq r \leq 100$

ab carré

ar "

br "



$a \wedge r = 1$

$$\begin{cases} (a+b)^2 = l^2 + (a-b)^2 \\ (b+r)^2 = m^2 + (r-b)^2 \\ (a+r)^2 = (m+l)^2 + (r-a)^2 \end{cases}$$

$$b = \frac{ar(a+r) \pm 2ar\sqrt{ar}}{(r-a)^2}$$

$l^2 = 4ab$

$m^2 = 4br$

$(m+l)^2 = 4ar = 4ab + 4br + 2ml$

~~$4(ar$~~ $16(ar - ab - br)^2 = 4 \times 16\, ab^2 r$

$(ar - ab - br)^2 = 4ab^2r \longrightarrow ar$: carré pf.

$a^2r^2 + (a+r)^2 b^2 - 2ar(a+r)b = 4arb^2$

$(r-a)^2 b^2 - 2ar(a+r)b + a^2r^2 = 0$

$\Delta' = a^2r^2(a+r)^2 - a^2r^2(r-a)^2 = 4a^3r^3$

(11)

$$(a-b)^2 r^2 - 2(a-b)abr + a^2b^2 = 4ab^2 r$$

$$(a-b)^2 r^2 - 2(a+b)abr + a^2b^2 = 0$$

$$\Delta' = a^2b^2(a+b)^2 - a^2b^2(a-b)^2$$
$$= 4a^2b^2ab = 4a^3b^3$$

$$r = \frac{(a+b)ab \pm 2ab\sqrt{ab}}{(a-b)^2} = \frac{ab}{(a-b)^2}\left[a+b \pm 2\sqrt{ab}\right]$$

$$\cancel{b=1,\ a=k^2 \quad \frac{k^2}{(k^2-1)^2}\left[k^2+1 \pm 2k\right]}$$

$$a+b = k \qquad \sqrt{ab} = l \qquad 2l \leq k$$

$$\frac{l^2}{k^2-4l^2}\left[k \pm 2l\right] = \frac{l^2(k \pm 2l)}{(k-2l)(k+2l)} = \frac{l^2}{k \mp 2l} = r$$

$$\cancel{\Rightarrow \boxed{k \mid l} \quad a^2+b^2 \mid ab} \qquad k \pm 2l \mid l^2$$

$$\frac{l^2}{k \pm 2l} \geq l \qquad kl \pm 2l^2 \leq l^2$$

$$\frac{l^2}{k+2l} \geq l \Rightarrow l^2 \geq kl + 2l^2 \Rightarrow \text{contr.}$$

$$\boxed{r = \frac{l^2}{k-2l}} \qquad \frac{l^2}{k-2l} \geq l \Rightarrow kl \leq 3l^2 \qquad k \leq 3l$$

$$\boxed{2l \leq k \leq 3l} \qquad l^2 \leq 100(k-2l)$$

$$\frac{l^2}{k-2l} \geq \frac{k}{2} \qquad l^2 \geq \frac{k^2}{2} - kl$$

$$k^2 \leq 2l(k+l)$$

$$\frac{k}{l} \leq 2\left(1 + \frac{l}{k}\right)$$

11

$ab = l^2$
$a+b = k$

$$x^2 - kx + l^2 = 0$$

$\Delta = k^2 - 4l^2$ carrée.

$k \wedge l = 1$

$l = 2 \quad k = 5 :$ $\quad 4, 1, \cancel{4}\, 4$
$l = 3 \quad k = 10$
$l = 4 \quad k = 10 \quad r = \frac{16}{2} = 8 \quad$ ~~(a = 8)~~
$l = 5$
$l = 6 : \quad ab = 2 \times 2 \times 3 \times 3 \qquad b = 4,\ a = 9,\ k = 13$

$r = 36/1 = 36$

$4, 9, 36 \qquad r = ab$

$$\frac{l^2}{k - 2l} = l^2 \qquad \boxed{36}$$

$k - 2l = 1$

$a + b - 2\sqrt{ab} = 1$

$4ab = (a+b-1)^2$

$a^2 + b^2 - 2ab - 2a - 2b + 1 = 0$

$n^2 + p^2 - 2np = 1$

$p \neq 1 \quad \Delta' = n^2 \mp (n^2 \mp 1) = 1$

$(n-p)^2 = 1$