① ♡ = 1

$$\begin{array}{r} 31 \\ \times\ 31 \\ \hline 162 \end{array}$$

→ ~~1621~~

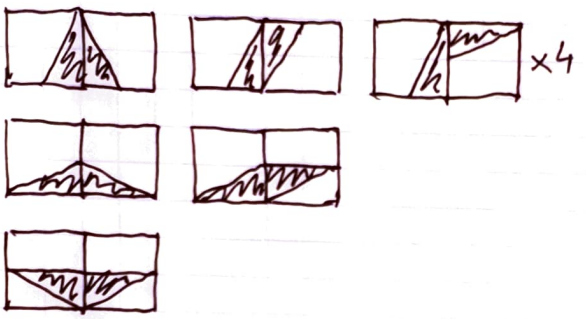
$$\begin{array}{r} 51 \\ \times\ 51 \\ \hline 102 \end{array}$$

→ 102

② A, B, C $A < B + C$

$4 + 3 + 3$
$4 + 2 + 4$ → $\underline{\underline{2}}$

③



→ 6
→ 2
→ 1

9

④ B : $10 \times \frac{30}{18}$ C : $10 \times \frac{30}{18} \times \frac{18}{25} = 10 \times \frac{6}{5} = 12$

⑤ 9 : 1 voisine 19. Idem 8, 7, …, 3.

0, 1, 2, 3, 4, 10, 11, 12, …, 21

3 - 13 - 10 - 0 - 20 - 21 - 11 - 12 - 15 - 16 - 17 - 18 - 19 - 14 - 4

2 1

→ 17

⑥

$$\begin{array}{r} a\,b \\ \times\ \boxed{7} \\ = \boxed{9\ \ \ } \\ c \end{array} \qquad \begin{array}{r|l} 900 & 7 \\ 20 & 128 \\ 60 & \end{array} \qquad \begin{array}{r|l} 986 & 7 \\ 28 & 140 \\ 06 & \end{array}$$

$\rightarrow a = 2$ ou 3

(b,c) : $(2,4)$ $(4,8)$ $(6,2)$ $(8,6)$

$$\begin{array}{r} 132 \\ \times\ 7 \\ = 94 \end{array} \qquad \begin{array}{r} 124 \\ \times\ 7 \\ = 968 \end{array} \qquad \begin{array}{r} 134 \\ \times\ 7 \\ 98 \end{array} \qquad \boxed{\begin{array}{r} 136 \\ \times\ 7 \\ = 952 \end{array}} \qquad \begin{array}{r} 138 \\ \times\ 7 \\ = 966 \end{array}$$

$\rightarrow$ 4 et 8

⑦ $21 - 6 - 2 = 13 = 4 + 9$ ou $\cancel{5+8}$

$21 - 6 = 15 = 7 + 8$

$21 - 4 - 8 = 9$ imp. $\Rightarrow$ 9 en haut.

$21 - 9 = 12 = 5 + 7$

↳ en bas

9

4 3

5

2 1

6 7 8

⑧ (12, 21) / 01 / 20

11 / 02 / 20

23 / 03 / 20
24 / 04 / 20
⋮
29 / 09 / 20 } 7

$\rightarrow$ ~~14~~ 16

(12, 21) / 10 / 20

(02, 20) / 11 / 20

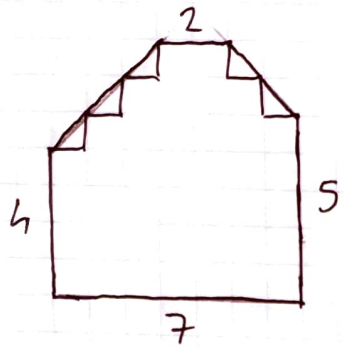
(01, 10) / 12 / 20

(9) $32 = 4 + 2a + 3b$ $\quad$ b?

$4 + a + b = 15 \rightarrow a = 11 - b$

$28 = \cancel{2a}\ 22 - 2b + 3b \rightarrow b = 6$ (1 sol°)

(10)



$2 + 4 + 6 + 4 \times 7 + 3 = \underline{\underline{43}}$

(11) 15 animaux $\quad$ $2c + d = 17$ $\quad$ $c \geq 2$ $\quad$ $d \geq 2$

$c + d \leq 13$

c	d
7	3
6	5
5	7
4	9

4 sol°: 3, 5, 7, 9

(12) Si $A = 2$ → imp. $A = 1$

$1BCD + 1BC + 1B = 2021$ $\quad$ $B < 9 \rightarrow B = 8$

$18CD + 18C + 18 = 2021$

$CD + C = 221 - 198 = 23 \rightarrow C = 2,\ D = 1$

$ABCD = 1821$ (1 sol°)

(13) $a \times a \times h$

$a^2 h = 2a^2 + 4ah = 2a(a+2h)$

$ah = 2(a+2h) = 2a + 4h$

$a = \dfrac{4h}{h-2} = 4 + \dfrac{8}{h-2}$

h	a
3	12
4	8
~~6~~	~~6~~
10	5

$\rightarrow$ 3 sol° : 5, 8, 12

(14) $(12+x)(21+x) = 2021 + x$

$x^2 + 33x + 252 = 2021 + x$

$x^2 + 32x - 1769 = 0$

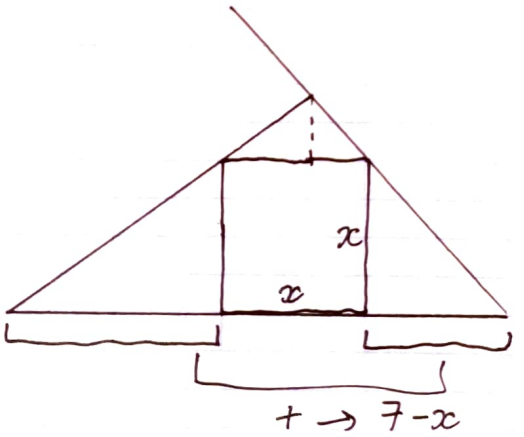
$\Delta' = 256 + 1769 = 2021 + 4 = 45^2$

$x = -16 \pm 45 = \boxed{-61 \text{ ou } 29} \rightarrow$ 2 sol°

Vérif : $49 \times 40 = 1960$ OK

$41 \times 50 = 2050$ OK

(15)



$\dfrac{x}{7-x} = \dfrac{h-x}{x}$

$h^2 + a^2 = 7{,}5^2$

$h^2 + (7-a)^2 = 6{,}5^2$

$a^2 - (7-a)^2 = 7{,}5^2 - 6{,}5^2$

$7(2a-7) = 14$

$2a - 7 = 2 \quad a = 4{,}5$

$h^2 = 7{,}5^2 - 4{,}5^2 = 3 \times 12 = 6^2$

$h = 6$

$(x-7)(x-6) = x^2 \qquad -13x + 42 = 0$

$x = 42/13$

$\rightarrow 3{,}23$ m

42,00	13
30	3,23
40	
1	

(16) $+ \to 0 \quad - \to 1 \quad \mathbb{Z}/2\mathbb{Z}$

1	1	
0 1	1	4
1 1 0	2	
1 0 1 1	3	
0 1 1 0 1	3	10
1 1 0 1 1 0	4	
1 0 1 1 0 1 1	5	
0 1 1 0 1 1 0 1	5	16
1 1 0 1 1 0 1 1 0	6	
1 0 1 1 0 1 1 0 1 1	7	22

0

4 10 16 22

6 6 6

0 0

$4n + 3n(n-1) = 3n^2 + n = n(3n+1)$

$n = 7 \to 7 \times 22 = 154$

(17) $(a, b, 180-a-b)$

$a \leqslant b \leqslant 180-a-b$

$a=1$:	$b = 1$ à 89	89
$a=2$:	$b = 2$ à 89	88
$a=3$:	$b = 3$ à 88	86
$a=4$:	$b = 4$ à 88	85
	⋮	⋮
$a=59$:	$b = 59$ à 60	2
$a=60$:	$b = 60$ à 60	1

$$\frac{89 \times 90}{2} - 3 \times \frac{29 \times 30}{2} = \frac{8010}{2} - 3 \times \frac{870}{2}$$

$$= \frac{8010 - 2610}{2} = \frac{5400}{2} = 2700$$

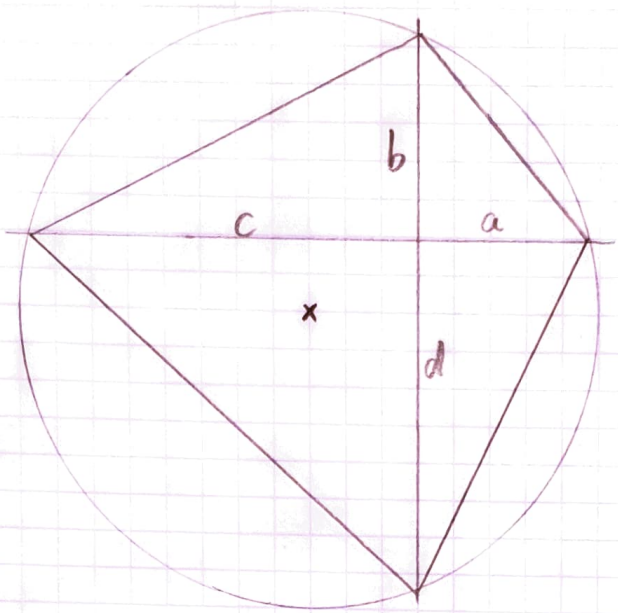
0 3 9 15 21

6 6 6

$3n + 3n(n-1) = 3n^2$

$n = 30 \to 3 \times 30^2 = 2700$ OK

18



$bd = ac$

$a = b$ et $c = d$?

$a\sqrt{2}$ et $c\sqrt{2}$ et $\sqrt{a^2+c^2}$

~~bd=ac~~